GARCH-M Model

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In finance, the return of a security may depend on its volatility (risk). To model such phenomena, the GARCH-in-mean (GARCH-M) model adds a heteroskedasticity term into the mean equation. It has the specification: $$x_t = \mu + \lambda \sigma_t + a_t$$ $$\sigma_t^2 = \alpha_o + \sum_{i=1}^p {\alpha_i a_{t-i}^2}+\sum_{j=1}^q{\beta_j \sigma_{t-j}^2}$$ $$a_t = \sigma_t \times \epsilon_t$$ $$\epsilon_t \sim P_{\nu}(0,1)$$

- Where:
- x_t is the time series value at time t.
- μ is the mean of GARCH model.
- λ is the volatility coefficient (risk premium) for the mean.
- a_t is the model's residual at time t.
- σ_t is the conditional standard deviation (i.e. volatility) at time t.
- p is the order of the ARCH component model.
- $\alpha_o,\alpha_1,\alpha_2,...,\alpha_p$ are the parameters of the the ARCH component model.
- q is the order of the GARCH component model.
- $\beta_1,\beta_2,...,\beta_q$ are the parameters of the the GARCH component model.
- $\left[\epsilon_t\right]$ are the standardized residuals: $$\left[\epsilon_t \right]\sim i.i.d$$ $$E\left[\epsilon_t\right]=0$$ $$\mathit{VAR}\left[\epsilon_t\right]=1$$
- P_{ν} is the probability distribution function for ϵ_t. Currently, the following distributions are supported:
    1. Normal distribution $$P_{\nu} = N(0,1)$$
    2. Student's t-distribution $$P_{\nu} = t_{\nu}(0,1)$$ $$\nu \succ 4$$
    3. Generalized error distribution (GED) $$P_{\nu} = \mathit{GED}_{\nu}(0,1)$$ $$\nu \succ 1$$

Remarks

1. A positive risk-premium (i.e. λ) indicates that data series is positively related to its volatility.
2. Furthermore, the GARCH-M model implies that there are serial correlations in the data series itself which were introduced by those in the volatility σ_t^2 process.
3. The mere existence of risk-premium is, therefore, another reason that some historical stocks returns exhibit serial correlations.

See Also

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