SARIMA

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The SARIMA model is an extension of the ARIMA model, often used when we suspect a model may have a seasonal effect.

By definition, the seasonal auto-regressive integrated moving average - SARIMA(p,d,q)(P,D,Q)s - process is a multiplicative of two ARMA processes of the differenced time series.

$$(1-\sum_{i=1}^p {\phi_i L^i})(1-\sum_{j=1}^P {\Phi_j L^{j \times s}})(1-L)^d (1-L^s)^D x_t = (1+\sum_{i=1}^q {\theta_i L^i})(1+\sum_{j=1}^Q {\Theta_j L^{j \times s}}) a_t$$ $$y_t = (1-L)^d (1-L^s)^D x_t$$

Where:

- x_t is the original non-stationary output at time t.
- y_t is the differenced (stationary) output at time t.
- d is the non-seasonal integration order of the time series.
- p is the order of the non-seasonal AR component.
- P is the order of the seasonal AR component.
- q is the order of the non-seasonal MA component.
- Q is the order of the seasonal MA component.
- s is the seasonal length.
- D is the seasonal integration order of the time series.
- a_t is the innovation, shock or the error term at time t.
- $\{a_t\}$ time series observations are independent and identically distributed (i.e. i.i.d) and follow a Gaussian distribution (i.e. $\Phi(0,\sigma^2)$)

Assuming y_t follows a stationary process with a long-run mean of μ, then taking the expectation from both sides, we can express ϕ_o as follows:

$$\phi_o = (1-\phi_1-\phi_2-\cdots-\phi_p)(1-\Phi_1-\Phi_2-\cdots-\Phi_P)$$

Thus, the SARIMA(p,d,q)(P,D,Q)s process can now be expressed as:

$$ (1-\sum_{i=1}^p {\phi_i L^i})(1-\sum_{j=1}^P {\Phi_j L^{j \times s}}) (y_t -\mu) = (1+\sum_{i=1}^q {\theta_i L^i})(1+\sum_{j=1}^Q {\Theta_j L^{j \times s}}) a_t$$ $$z_t=y_t-\mu$$ $$(1-\sum_{i=1}^p {\phi_i L^i})(1-\sum_{j=1}^P {\Phi_j L^{j \times s}}) z_t = (1+\sum_{i=1}^q {\theta_i L^i})(1+\sum_{j=1}^Q {\Theta_j L^{j \times s}}) a_t$$

In sum, z_t is the differenced signal after we subtract its long-run average.

Notes: The order of the seasonal or non-seasonal AR (or MA) component is solely determined by the order of the last lagged variable with a non-zero coefficient. In principle, you can have fewer parameters than the order of the component.

Remarks

1. The variance of the shocks is constant or time-invariant.
2. The order of the seasonal or non-seasonal AR (or MA) component is solely determined by the order of the last lagged variable with a non-zero coefficient. In principle, you can have fewer parameters than the order of the component.
3. **Example:** Consider the following SARIMA(0,1,1)(0,1,1)12 process: $$(1-L)(1-L^{12})x_t-\mu = (1+\theta L)(1+\Theta L^{12})a_t$$ **Note:** This is the AIRLINE model, a special case of the SARIMA model.

Requirements

Header	SFSDK.H
Library	SFSDK.LIB
DLL	SFSDK.DLL

References

Hamilton, J .D.; Time Series Analysis , Princeton University Press (1994), ISBN 0-691-04289-6

Tsay, Ruey S.; Analysis of Financial Time Series John Wiley & SONS. (2005), ISBN 0-471-690740

See Also

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