

ARIMA

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The ARIMA model is an extension of the ARMA model that applies to non-stationary time series (the kind of time series with one or more integrated unit-roots). By definition, the auto-regressive integrated moving average (ARIMA) process is an ARMA process for the differenced time series:

$$\left[(1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p)(1-L)^d x_t - \phi_0 = (1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q) a_t \right] \quad y_t = (1-L)^d x_t$$

Where:

- x_t is the original non-stationary output at time t .
- y_t is the observed differenced (stationary) output at time t .
- d is the integration order of the time series.
- a_t is the innovation, shock or error term at time t .
- p is the order of the last lagged variables.
- q is the order of the last lagged innovation or shock.
- $\{a_t\}$ time series observations are independent and identically distributed (i.e. i.i.d) and follow a Gaussian distribution (i.e. $\Phi(0, \sigma^2)$)

Remarks

1. The variance of the shocks is constant or time-invariant.
2. Assuming y_t (i.e. $(1-L)^d x_t$) is a stationary process with a long-run mean of μ , then taking the expectation from both sides, we can express ϕ_0 as follows: $\phi_0 = (1 - \phi_1 - \phi_2 - \dots - \phi_p)\mu$
3. Thus, the ARIMA(p, d, q) process can now be expressed as: $(1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p)(y_t - \mu) = (1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q) a_t$
 $z_t = y_t - \mu$
 $(1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p) z_t = (1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q) a_t$
4. In sum, z_t is the differenced signal after we subtract its long-run average.
5. The order of an ARIMA process is solely determined by the order of the last lagged variable with a non-zero coefficient. In principle, you can have fewer number of parameters than the order of the model.
6. **Example:** Consider the following ARIMA(12,2) process:

$$(1 - \phi_1 L - \phi_{12} L^{12}) (y_t - \mu) = (1 + \theta_2 L^2) a_t$$

Requirements

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References

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Tsay, Ruey S.; [Analysis of Financial Time Series](#) John Wiley & SONS. (2005), ISBN 0-471-690740

See Also

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